$z^2+5z+6 \,\big)\, z^2-1$

z^2+5z+6

$-5z$

VIT

Vellore Institute of Technology

Reg. No. : 21BCE1446

Final Assessment Test (FAT) - November/December 2022

Programme	B.Tech.	Semester	Fall Semester 2022-23
Course Title	COMPLEX VARIABLES AND LINEAR ALGEBRA	Course Code	BMAT201L
Faculty Name	Prof. Dr.Durga Nagarajan	Slot	A1+TA1+TAA1
		Class Nbr	CH2022231001160
Time	3 Hours	Max. Marks	100

Part-A (10 X 10 Marks)

Answer any 10 questions

1. Determine the analytic function $f(z) = u + iv$ given that $u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$ and $f(\frac{\pi}{2}) = 0$. [10]

2. Find the image of the rectangle with vertices $-1+i, 1+i, 1+2i$ and $-1+2i$ under the linear mapping $f(z) = 4iz + 2 + 3i$. [10]
   Sketch the rectangle and its image.

3. a) Find the bilinear transformation which maps the points $z = i, z = -1$ and $z = 1$ into the points $w = 0, w = 1$ and $w = \infty$. [10]
   b) Find the fixed points and image of the interior of the circle $|z| = 1$ under the transformation $w = \frac{z-i}{1-iz}$.

4. Using Contour integration, evaluate the real integral $\int_0^{2\pi} \frac{1+2\cos\theta}{10+8\cos\theta}\, d\theta$. [10]

5. a) Find Taylor's series expansion to represent $\frac{z^2-1}{(z+2)(z+3)}$ in $|z| = 2$. [10]
   b) Find the nature of singularity and find the residue for i) $f(z) = \frac{z-\sin z}{z^3}$ ii) $f(z) = \frac{1-\cos z}{z}$.

6. Find a basis for the row space and null space of $A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 0 \\ 2 & 1 & 5 & -2 \end{pmatrix}$. [10]

7. Let V be the vector space of functions $f : \mathbb{R} \to \mathbb{R}$. Verify whether the following are subspaces of V or not. Justify your answer. [10]
   i) $W_1 = \{f \in V : f(1) = 3\}$.
   ii) $W_2 = \{f \in V : f(3) = f(1)\}$.
   iii) $W_3 = \{f \in V : f(-x) = -f(x)\}$.

8. A mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by [10]
   $$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 + x_2 + 2x_3, x_1 + 2x_2 + x_3), \quad x_1, x_2, x_3 \in \mathbb{R}.$$
   Show that T is a linear mapping. Find Ker T and the dimension of Ker T.

9. Let $S = \{v_1 = (1,2,0), v_2 = (1,3,2), v_3 = (0,1,3)\}$ and $S' = \{u_1 = (1,2,1), u_2 = (0,1,2), u_3 = (1,4,6)\}$, [10]
   i) Find the change of basis matrix P from S to S',
   ii) Find the change of basis matrix Q from S' to S,
   iii) verify $Q = P^{-1}$.

10. Let $\mathbb{R}^3$ have the Euclidean inner product. Use the Gram Schmidt process to transfer the basis $\{u, v, w\}$ into an orthonormal basis, where $u = (1, 0, -1), v = (-7, 4, -2), w = (-3, 0, -1)$. [10]

$-2 - \frac{9}{2} + 8 \quad \frac{-4-9+16}{2} =$

$$\frac{+6 \pm \sqrt{36-16}}{+2}$$

11. Solve the following system, by using Gauss elimination method [10]

$2y - z = 1,$

$4x - 10y + 3z = 5,$

$3x - 3y = 6.$

$-4\lambda^2 - 4\lambda - 1$

$$\frac{4 \pm \sqrt{16-16}}{-2 \times 4} \quad -1/2$$

12. [10]

Let A and B be two matrices such that $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$, $B = I - \frac{1}{2}A$. If λ_i and λ_i' are the eigenvalues of A and B respectively. Find $\lambda_i, \lambda_i', i = 1, 2, 3$, hence verify that $\lambda_i' + \frac{1}{2}\lambda_i = 1$

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$-4\lambda^2 - 4\lambda$

$\lambda - 1 \,\big)\, -4\lambda^3 + 3\lambda + 1$

$-4\lambda^3 + 4\lambda^2$

$-4\lambda^2 + 3\lambda + 1$

$-4\lambda^2 + 4\lambda$

$-4\lambda^2 - 8\lambda - 2 \quad -\lambda + 1$

$8 + 2\lambda^2 - 8\lambda$

$-4\lambda - \lambda^3 + 4\lambda^2 + \lambda$

$-\lambda^3 + 6\lambda^2 - 9\lambda$

0 .

$-8 + 24 - 18$

$2\lambda - 6$

$(\lambda - 3)$

$(\lambda - 3)$

$k \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$

$1(\lambda - 2 - 1)$

$(3 - \lambda) -$

$(-1)(1 + 2 - \lambda)$

$x = 2 - k$

$y = k$

$z = -1 + 2k$

$-8 + 24 - 18 + 3$

6

-207
26
-181
69
-112

$4 + \lambda^2 - 4\lambda$

12

$-27 + 56 - 27 + 3$

-54

2

2

6

$-30 + 12$

-18

3

$\frac{1}{2} + \frac{3}{2}$

$\frac{6 - 3k}{3}$

-69
-69
-138
91
-47

$2k - 1$

$2 - k$

9 :

37

$15 - 24$

-9

$-\lambda$

$2k - 1 =$

$y = \frac{1 + k}{2}$