VIT Vellore Institute of Technology

Reg. No. : 22BEC1022

Final Assessment Test (FAT) – January/February 2023

Programme	**B.Tech.**	Semester	**Fall Semester 2022-23**
Course Title	**CALCULUS**	Course Code	**BMAT101L**
Faculty Name	**Prof. Manoj Kumar Singh**	Slot	**B1+TB1**
		Class Nbr	**CH2022231700260**
Time	**3 Hours**	Max. Marks	**100**

Part A (10 X 10 Marks)
Answer any 10 questions

1. Find the area between the circle $x^2+y^2=2ax$ and parabola $y^2=ax$. **[10]**
2. (a) If $0<a<b<1$, then prove that, $\frac{(b-a)}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{(b-a)}{1+a^2}$, hence show that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\left(\frac{4}{3}\right)<\frac{\pi}{4}+\frac{1}{6}$. (5 Marks) **[10]**
   (b) Find $\frac{\partial z}{\partial x}$, if $yz-\ln z=x+y$, where z is a function of two independent variables $x\&y$ and prove that the partial derivative exists. (5 Marks)
3. Show that the function defined by $f(x,y)=\begin{cases}\frac{2xy}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$ is continuous at every point except the origin. **[10]**
4. Find the points on the surface $x^2-zy=4$ closest to the origin. **[10]**
5. (a) Using Taylor's Formula for $f(x,y)$ at the origin, find the cubic approximation of $f(x,y)=\frac{1}{1-x-y}$. (5 Marks) **[10]**

   (b)Evaluate $\int_0^\infty \frac{x^a}{a^x}dx\ (a>0)$. (5 Marks)
6. (a) Find the value of integral $I=\int_0^8\int_{y^{\frac{1}{3}}}^2\sqrt{x^4+1}\,dx\,dy$ by changing the order of integration. (5 Marks) **[10]**

   (b) Find the volume of the solid region bounded by the paraboloid $az=x^2+y^2$ and cylinder $x^2+y^2=b^2$. (5 Marks)
7. Evaluate $\int_0^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-(x^2+y^2)}^{(x^2+y^2)}21xy^2dz\,dy\,dx.$ **[10]**
8. (a) Obtain the area of the ellipse with semi-major axis a and semi-minor axis as b respectively using gamma function. (4 Marks) **[10]**
   (b) Evaluate the Dirichlet integral $\iiint_D x^{l-1}y^{m-1}z^{n-1}dx\,dy\,dz$, where D is the region bounded by the first octant and $\left(\frac{x}{a}\right)^p+\left(\frac{y}{b}\right)^q+\left(\frac{z}{c}\right)^r\leq 1$. (6 Marks)
9. (a) Find the divergence of $r^n\vec{r}$, where $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ and $r=|\vec{r}|$. Also find the value of n for which $r^n\vec{r}$ is a solenoidal field. (4 Marks) **[10]**
   (b) Find the directional derivative of the scalar field $\phi=x^3-5x^2y-z$ at $P(1,1,1)$ in the direction of the line $\frac{x-1}{2}=\frac{y+1}{-2}=z$. In what directions does ϕ changes most rapidly at P and what are the rates of change in these directions. (6 Marks)
10. Prove the following vector identities **[10]**

(a) $div(curl(\vec{F})) = 0$ (3 Marks)

(b) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ and $r = |\vec{r}|$. (7 Marks)

11. (a) Using Green's Theorem evaluate $\oint_C \left[(2xy - x^2)\,dx + (x^2 + y^2)\,dy\right]$ where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$. (5 Marks) [10]

(b) If $\vec{F} = (4xy - 3x^2z^2)\,\vec{i} + 2x^2\,\vec{j} - 2x^3z\,\vec{k}$, check whether the integral $\oint_C \vec{F} \cdot \vec{dr}$ is independent of the path C. (5 Marks)

12. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\,\vec{i} - yz^2\,\vec{j} - y^2z\,\vec{k}$ over the upper half surface $x^2 + y^2 + z^2 = 1$, bounded by its projections on the xy-plane. [10]

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